1 Sept 2009

Section 1.2

Definition of limit

The limit of f(x) as x approaches c is L

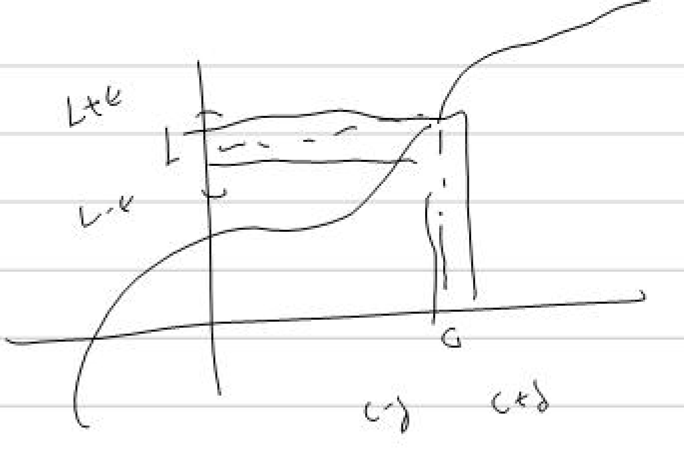
written $\lim_{x \to c} f(x) = L$

if and only if

f is defined for all x in some open interval containing c except possibly at c itself

And for any $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - c| < \delta$

$\lim_{x \to 0} \sqrt{x} = \text{DNE}$



Prove $\lim_{x \to 3} (2x-1) = 5$

Let $y = 2x-1$, since y is defined for all $\mathbb{R}$, it is defined for some open interval containing 3,

For any $\epsilon > 0$ let $\delta = \frac{\epsilon}{2}$

Whenever $0 < |x-3| < \delta$

$$|x-3| < \frac{\epsilon}{2}$$

$$2|x-3| < \epsilon$$

$$|2| \cdot |x-3| < \epsilon$$

$$|2x-6| < \epsilon$$

$$|(2x-1)-5| < \epsilon$$

$\therefore$ $\lim_{x \to 3} (2x-1) = 5$

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pre-proof

$$|(2x-1)-5| < \epsilon$$

$$|2x-6| < \epsilon$$

$$2|x-3| < \epsilon$$

$$|x-3| < \frac{\epsilon}{2}$$

Prove $\lim_{x \to -2} (x^2 + 1) = 5$

Since $y = x^2 + 1$ is defined for all $\mathbb{R}$, it is defined for all x in some open interval containing -2,

For any $\epsilon > 0$ let $\delta = \min\left(\frac{\epsilon}{5}, 1\right)$
whenever $0 < |x - -2| < \delta$

$$|x+2| < \frac{\epsilon}{|x-2|}$$

$$|x-2|\,|x+2| < \epsilon$$

$$|x^2 - 4| < \epsilon$$

$$|(x^2+1) - 5| < \epsilon$$

$$\therefore \lim_{x \to -2} (x^2+1) = 5$$

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pre proof

(not my proof)

$$|(x^2+1) - 5| < \epsilon$$

$$|x^2 - 4| < \epsilon$$

$$|(x+2)(x-2)| < \epsilon$$

$$\frac{\epsilon}{5} < \frac{\epsilon}{|x-2|} < \frac{\epsilon}{3}$$

$$|x+2| < \frac{\epsilon}{|x-2|}$$

$$\delta = \min\left(\frac{\epsilon}{5}, 1\right)$$

Scaler Law

$$\lim_{x \to c} K f(x) = K \lim_{x \to c} f(x)$$

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or if $\lim_{x \to c} f(x) = L$

then $\lim_{x \to c} K f(x) = KL$

$\lim_{x \to c} f(x) = L$ given

$\therefore$ f must defined for all x on some open interval containing c except possibly at c itself $\therefore$ $y = K f(x)$ is defined for the same interval.

Since $\lim_{x \to c} f(x) = L$ then there exist

some $\delta > 0$ for any $\epsilon > 0$ such that

$|f(x) - L| < \epsilon$ whenever $0 < |x - c| < \delta$

$\therefore$ for and $\epsilon > 0$ such as $\frac{\epsilon}{|K|}$ there exist a $\delta > 0$ such that whenever $0 < |x - c| < \delta$ then $|f(x) - L| < \frac{\epsilon}{|K|}$

$\therefore$ $|K| \, |f(x) - L| < \epsilon$

$|K f(x) - KL| < \epsilon$