4 Sept 2009

Almost = thm

$\lim\limits_{x\to c}$ ... $f(x) = g(x)$ for all x on some open interval containing c except at c itself

and $\lim\limits_{x\to c} g(x) = L$

then $\lim\limits_{x\to c} f(x) = L$

proof

Since $\lim\limits_{x\to c} g(x) = L$

then g is defined for all x on some open interval containing c except possibly at c itself

Since $f(x) = g(x)$ for $x \neq c$ on some open interval then $f(x)$ must be defined for all x on some open interval containing c except possibly at c itself

for any $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x-c| < \delta$ then $|g(x) - L| < \epsilon$

Since $x \neq c$ then $|f(x) - L| < \epsilon$

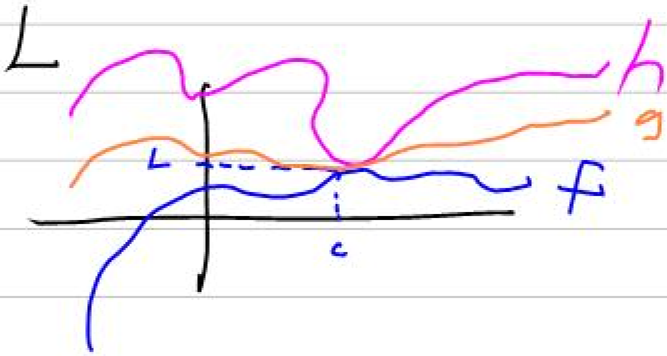
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Squeeze thm

if $f(x) \le g(x) \le h(x)$ for all x on some open interval containing c except possibly at c itself

And $\lim_{x \to c} f(x) = L$ And $\lim_{x \to c} h(x) = L$

then $\lim_{x \to c} g(x) = L$



if $0 < |x - c| < \delta_1$ then $|f(x) - L| < \epsilon$

$0 < |x - c| < \delta_2$ then $|h(x) - L| < \epsilon$

for any $\epsilon > 0$ let $\delta = \min(\delta_1, \delta_2)$

whenever $0 < |x - c| < \delta$

therefore $|f(x) - L| < \epsilon$ And $|h(x) - L| < \epsilon$

$-\epsilon < f(x) - L < \epsilon$ $\quad -\epsilon < h(x) - L < \epsilon$

$L - \epsilon < f(x) < L + \epsilon$ $\quad L - \epsilon < h(x) < \epsilon + L$

$L - \epsilon < f(x) \le g(x) \le h(x) < \epsilon + L$

$$L-\epsilon < f(x) \le g(x) \le h(x) < L+\epsilon$$

$$L-\epsilon < g(x) < L+\epsilon$$

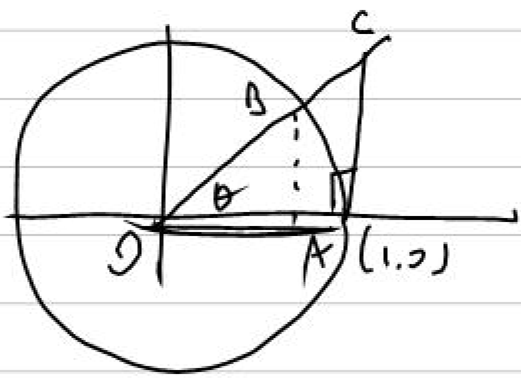
$$-\epsilon < g(x)-L < \epsilon$$

$$|g(x)-L| < \epsilon$$

$$\text{i.e.} \quad \lim_{x\to c} g(x) = L$$

$$4-x^2 \le f(x) \le 4+x^2$$

$$\lim_{x\to 0} f(x) = 4$$



$B(\cos\theta, \sin\theta)$

$C(1, \tan\theta)$

Area $\triangle BOA$ $\le$ Area of sector BOA $\le$ Area of $\triangle COA$

$$\tfrac{1}{2}(1)(\sin\theta) \le \frac{\theta}{2} \le \tfrac{1}{2}(1)\tan\theta$$

$$\sin\theta \le \theta \le \frac{\sin\theta}{\cos\theta}$$

$$1 \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}$$

$$1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta$$