8 Sept 2009

78) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 2x}{\sin 3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{\sin 2x}{x}}{\frac{\sin 3x}{x}}$

$= \dfrac{2\lim_{2x\to 0} \frac{\sin 2x}{2x}}{3\lim_{3x\to 0} \frac{\sin 3x}{3x}} = \frac{2(1)}{3(1)} = \frac{2}{3}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

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72) $\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{x\cos^2 x} =$

$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{\cos^2 x}$

$1 \cdot 0 = 0$

$\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$

$= \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{1+\cos x}$

$1 \cdot \frac{0}{2} = 0$

$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x} = 0}$

#105

$$\lim_{x\to 0} \frac{1}{x} = DNE \qquad \lim_{x\to 0} \frac{-1}{x} = DNE$$

$$\lim_{x\to 0} \left(\frac{1}{x} + \frac{-1}{x}\right) = 0$$

Notes

Def of continuity at a point

f(x) is continuous at x = c
if and only if

i) f(c) exists

ii) $\lim_{x\to c} f(x)$ exists

iii) $\lim_{x\to c} f(x) = f(c)$

---

2 types of Discontinuity

Removable (if discont. but $\lim_{x\to c} f(x)$ exists)

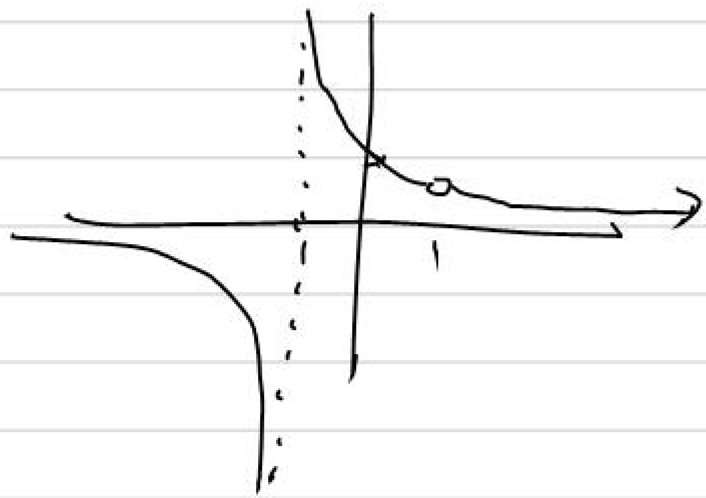
Essential (Non-removable)

. $f(x) = \frac{x-1}{x^2-1}$ discont at $x=1$ (removable) and $x=-1$ essential or non-removable

$y = \frac{x-1}{x^2-1}$

$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$ $\therefore$ disc. is removable

$\lim_{x \to -1} \frac{x-1}{x^2-1} = DNE$ $\therefore$ disc is essential



Def. The limit of $f(x)$ as x approaches c from the right (left) is L

written $\lim_{x \to c^{+}} f(x) = L$ iff f is

defined for all x on some open interval (c, b) (b, c)

and for any $\epsilon > 0$ there exist a $\delta > 0$

such that $|f(x) - L| < \epsilon$

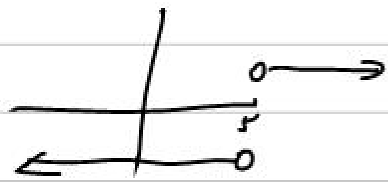
whenever

$0 < x - c < \delta$

Red for left handed limit

$0 < c - x < \delta$

$$\lim_{x\to 5} \frac{|x-5|}{x-5} = DNE$$



$$\lim_{x\to 5^+} \frac{|x-5|}{x-5} = 1$$

$$\lim_{x\to 5^-} \frac{|x-5|}{x-5} = -1$$

$\lim_{x\to c} f(x) = L$ iff $\lim_{x\to c^+} f(x) = L$

<u>And</u>

$\lim_{x\to c^-} f(x) = L$

Def of continuity on an <u>open</u> interval

f is continuous on (a,b) if f is continuous for all $c \in (a,b)$

Def of continuity on a <u>closed</u> interval

f is cont on $[a,b]$ iff

f is cont on (a,b)

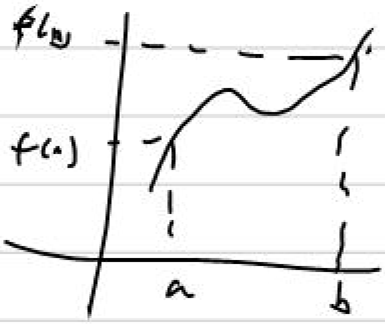
and $\lim_{x\to a^+} f(x)$ exists, $f(a)$ exists and $\lim_{x\to a^+} f(x) = f(a)$

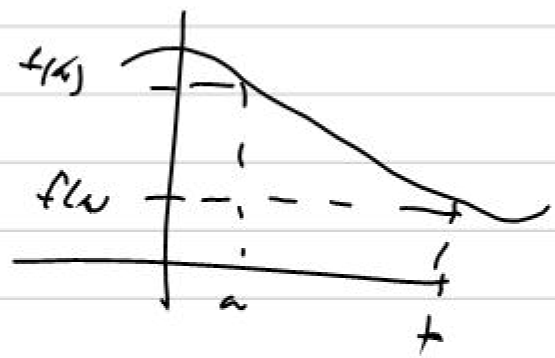
And $\lim_{x\to b^-} f(x)$ exists, $f(b)$ exists And $\lim_{x\to b^-} f(x) = f(b)$

$f(x) = \sqrt{x}$ $\quad [0,1]$

IVT (Intermediate Value Theorem)

if f is continuous on the closed interval $[a,b]$ and d is between $f(a)$ and $f(b)$ (i.e. $f(a) < d < f(b)$ or $f(b) < d < f(a)$) then there exist some $c \in (a,b)$ such that $f(c) = d$





$\tan \frac{\pi}{4} = 1$ $\qquad \tan \frac{3\pi}{4} = -1$