10 Sept 2009

p183 #107 $f(x) = \begin{cases} 1-x^2 & x \le c \\ x & x > c \end{cases}$

$\lim_{x \to c^+} f(x) = \lim_{x \to c^+} x = c$ $\quad \lim_{x \to c^-} f(x) = \lim_{x \to c^-} (1-x^2) = 1-c^2$

$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ $\quad 1-c^2 = c$

$\Rightarrow c^2 + c - 1$

$c = \frac{-1 \pm \sqrt{1+4}}{2}$

$$\boxed{c = \frac{-1 \pm \sqrt{5}}{2}}$$

#108 $(-\pi/2, \pi/2)$ $\quad \tan x = y$

$\lim_{x \to \pi/2^-} \tan x = \infty$ $\quad \lim_{x \to -\pi/2^+} \tan x = -\infty$

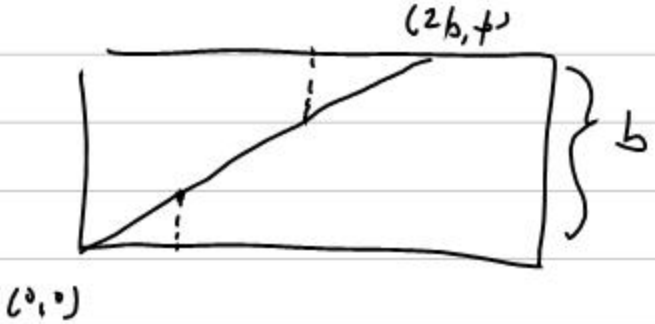
$\therefore \tan b = M > y$ $\quad \tan a = N < y$

~~Lem~~ $N < y < M$

$\tan a < y < \tan b$

$\therefore$ IVT there exists some $c \in (M, N)$ such that $f(c) = y$

(20)

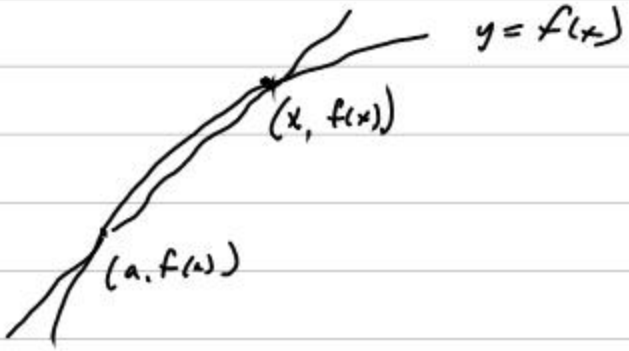


$\frac{b}{2b} = \frac{1}{2}$

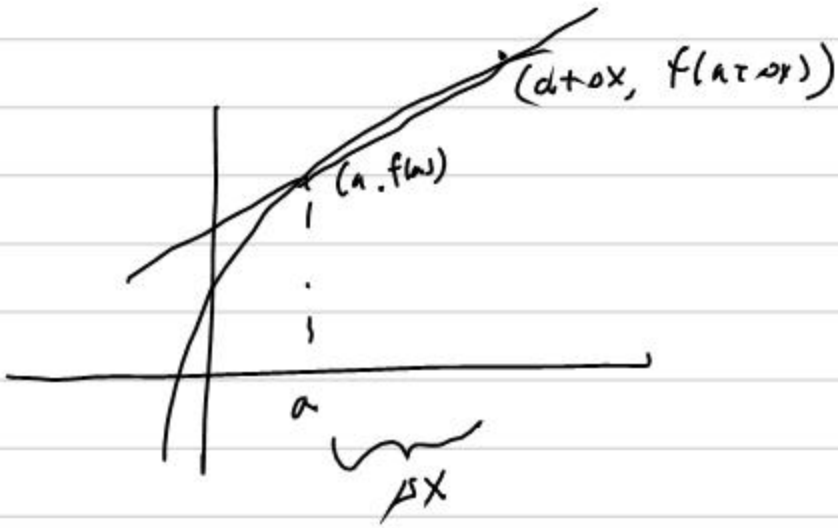
~~$f(x) = 2x$~~

a) $f(x) = \frac{1}{2}x$

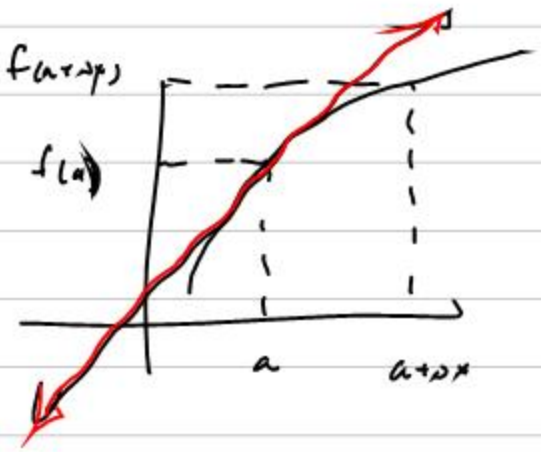
b) $g(x) = \begin{cases} \frac{1}{2}x & x \leq b \\ b - \frac{1}{2}x & x > b \end{cases}$



$$\frac{f(x) - f(a)}{x - a}$$



$$\frac{f(a+\Delta x) - f(a)}{\Delta x}$$



$$\lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$