16 Sept 2009

15) $f(x) = \frac{1}{\sqrt{x}}$ $\qquad x + 2y - 6 = 0$

$y = \frac{-x+6}{2}$

$$\lim_{x \to c} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{c}}}{x-c} = \lim_{x \to c} \frac{\sqrt{c} - \sqrt{x}}{\sqrt{xc}\,(x-c)} = \lim_{x \to c} \frac{(\sqrt{c}-\sqrt{x})(\sqrt{c}+\sqrt{x})}{\sqrt{xc}\,(x-c)(\sqrt{c}+\sqrt{x})}$$

$$= \lim_{x \to c} \frac{\overset{-1}{\cancel{c-x}}}{\sqrt{xc}\,\cancel{(x-c)}(\sqrt{c}+\sqrt{x})} = \lim_{x \to c} \frac{-1}{\sqrt{xc}\,(\sqrt{c}+\sqrt{x})}$$

$$= \frac{-1}{\sqrt{c^2}\,(\sqrt{c}+\sqrt{c})} = \frac{-1}{2c\sqrt{c}}$$

or

$$\lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{c+\Delta x}} - \frac{1}{\sqrt{c}}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{c} - \sqrt{c+\Delta x}}{\Delta x\,(\sqrt{c+\Delta x})\sqrt{c}} \cdot \frac{\sqrt{c} + \sqrt{c+\Delta x}}{\sqrt{c} + \sqrt{c+\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{c - (c+\Delta x)}{\Delta x\,(\sqrt{c+\Delta x})\sqrt{c}\,(\sqrt{c}+\sqrt{c+\Delta x})} =$$

$$= \lim_{\Delta x \to 0} \frac{-\overset{1}{\cancel{\Delta x}}}{\cancel{\Delta x}\,(\sqrt{c+\Delta x})\sqrt{c}\,(\sqrt{c}+\sqrt{c+\Delta x})} = \frac{-1}{2c\sqrt{c}}$$

$\frac{-1}{2c\sqrt{c}} = -\frac{1}{2}$ $\qquad c\sqrt{c} = 1$ $\qquad f(1) = \frac{1}{\sqrt{1}} = 1$

$c^{3/2} = 1$

$c = 1$

$$y - 1 = -\frac{1}{2}(x-1)$$

27) $f(x) = x^3$ $(2, 8)$

Power Rul

$$f'(2) = \lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$$

$$= \lim_{x \to 2} (x^2+2x+4) = 12$$

$$y - 8 = 12(x-2)$$

$$= \lim_{\Delta x \to 0} \frac{(2+\Delta x)^3 - 8}{\Delta x} = \lim_{\Delta x \to 0} \frac{8 + 12\Delta x + 6\Delta x^2 + \Delta x^3 - 8}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(12 + 6\Delta x + \Delta x^2)}{\cancel{\Delta x}} = 12$$

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

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41) $y = g(x)$ $(5, 2)$ passes through $(9, 0)$

$g(5) = 2$ $g'(5) = \frac{0-2}{9-5} = -\frac{2}{4} = -\frac{1}{2}$

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constant law

$D_x C = 0$ if $f(x) = c$ then $f'(x) = 0$

proof $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{C - C}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = \lim_{\Delta x \to 0} 0 = 0$$

Power Rule $\qquad D_x\, x^n = n x^{n-1}$

$$D_x\, x^n = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{x^n + n x^{n-1}\Delta x + \binom{n}{2} x^{n-2}\Delta x^2 + \dots + n x \Delta x^{n-1} + \Delta x^n - x^n}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left(n x^{n-1} + \binom{n}{2} x^{n-2}\Delta x + \binom{n}{3} x^{n-3}\Delta x^2 + \dots + n x \Delta x^{n-2} + \Delta x^{n-1}\right)}{\cancel{\Delta x}}$$

$$n x^{n-1}$$

$$D_x\, x^3 = 3x^2$$

$$D_x \frac{1}{\sqrt{x}} = D_x\, x^{-1/2} = -\tfrac{1}{2} x^{-1/2-1} = -\tfrac{1}{2} x^{-3/2} = \frac{-1}{2x\sqrt{x}}$$

The scaler rule $\qquad D_x\, k f(x) = k D_x f(x)$

or $\qquad y = k f(x) \quad$ then $\quad y' = k f'(x)$

$$D_x\, k f(x) = \lim_{\Delta x \to 0} \frac{k f(x+\Delta x) - k f(x)}{\Delta x} = k \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= k f'(x)$$

$D_x \; 5x^2 = 10x$

Sum and difference law

$$D_x \left(f(x) \pm g(x) \right) = D_x f(x) \pm D_x g(x)$$

$$D_x \left[f(x) + g(x) \right] = \lim_{\Delta x \to 0} \frac{\left[f(x+\Delta x) + g(x+\Delta x) \right] - \left[f(x) + g(x) \right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x) + g(x+\Delta x) - g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$f'(x) + g'(x)$$

$$y = 3x^4 - 7x^2 + 5x - 4$$

$$y' = 12x^3 - 14x + 5$$

Differentiability implies continuity

If f is differentiable at $x = c$
then f is continuous at $x = c$