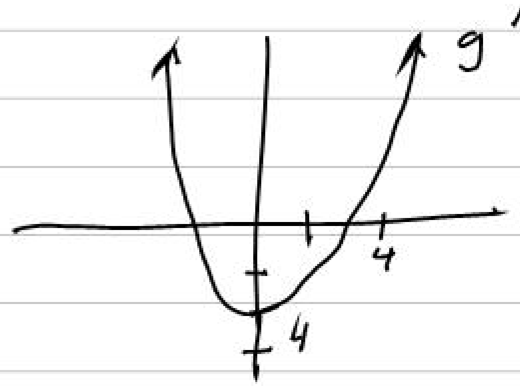
17 Sept 2009

#59 ~~[illegible]~~



a) $g'(0) = -3$

b) $g'(3) = 0$

c) $g'(1) = -8/3$

↳ g will have a tangent line slope of $-8/3$ at $x = 1$

d) $g'(-4) = 7/3$ g will have tangent line slope of $7/3$ at $x = -4$

e) ~~$g(-4)$~~ $g(6) - g(4) > 0$

103) $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

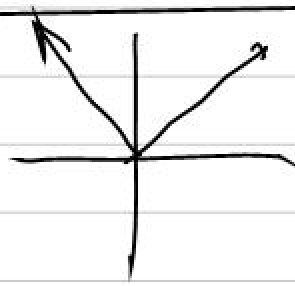
$g(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

proved on board

Differentiability implies continuity

$$y = \begin{cases} x \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$y = |x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$



$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \text{DNE}$$

$\therefore$ f is <u>Not</u> diff at $x = 0$

$$f'_+(0) = \lim_{x \to 0^+} \frac{x - 0}{x - 0} = 1$$

$$f'_-(0) = \lim_{x \to 0^-} \frac{-x - 0}{x - 0} = -1$$

SHARP turns or "cusps" cause Non-differentiability

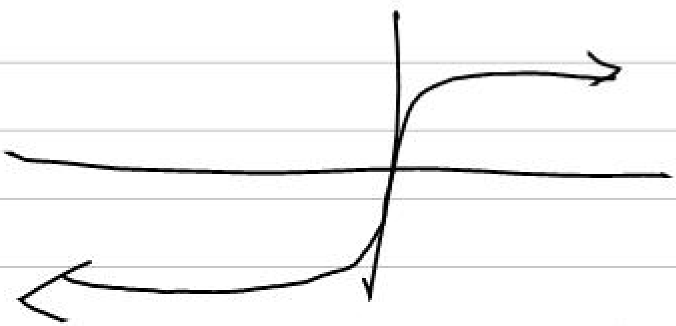
$y = \frac{1}{x}$ $\qquad f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ DNE

Discontinuity will cause non-differentiability

$y = \sqrt[3]{x}$

← bad drawing

$$f'(0) = \lim_{x \to 0} \frac{\sqrt[3]{x} - 0}{x - 0} = \lim_{x \to 0} \frac{1}{x^{2/3}} = \infty$$



Vertical tangents causes non-diff.

$$D_x[\sin x] = \lim_{\Delta x \to 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$\lim_{\Delta x \to 0}$ si

$y = \sin x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \left[\frac{\sin x (\cos \Delta x - 1)}{\Delta x} + \frac{\cos x \sin \Delta x}{\Delta x} \right]$$

$$\lim_{\Delta x \to 0} \sin x \lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} + \lim_{\Delta x \to 0} \cos x \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x}$$

$$(\sin x)(0) + (\cos x)(1) = \cos x$$

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proved similarly $\frac{d}{dx} \cos x = -\sin x$