~~21~~ Sept 2009

p. 126 36) $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

$$f(x) = (x^4 - x^3 + x^2 - x)(x^2 + x + 1)$$

$$f'(x) = (4x^3 - 3x^2 + 2x - 1)(x^2 + x + 1) +$$

$$(2x + 1)(x^4 - x^3 + x^2 - x)$$

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30) $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

$$f'(x) = \frac{1}{3}x^{-2/3}(\sqrt{x} + 3) + \frac{1}{2\sqrt{x}}(\sqrt[3]{x})$$

$$\frac{1}{3}x^{-2/3}(x^{1/2} + 3) + \frac{1}{2}x^{-1/2}(x^{1/3})$$

$$\frac{1}{3}X^{-1/6} + x^{-2/3} + \frac{1}{2}X^{-1/6}$$

$$\frac{1}{3}x^{-1/6} + X^{-4/6} + \frac{1}{2}x^{-1/6}$$

$$\frac{1}{6}x^{-4/6}(2x^{3/6} + 3x^{3/6} + 6)$$

$$\boxed{\frac{5\sqrt{x} + 6}{6x^{2/3}}}$$

57) $g(\theta) = \dfrac{\theta}{1-\sin\theta}$

$$g'(\theta) = \frac{1(1-\sin\theta) - (-\cos\theta)\theta}{(1-\sin\theta)^2}$$

$$g'(\theta) = \frac{1-\sin\theta + \theta\cos\theta}{(1-\sin\theta)^2}$$

60) $f(x) = \tan x \cot x$ $\qquad (1,1)$

$f'(x) = \sec^2 x \cot x - \csc^2 x \tan x$

$f'(1) = 0$ $\qquad \boxed{y=1}$

OR

$f(x) = \tan x \dfrac{1}{\tan x} = 1, \; x \neq k\pi/2$

$f'(x) = 0$

The chain Rule

If $y = f(u)$ is a diff function of u
and $u = g(x)$ is a diff function of x

then $y = f(g(x))$ is a diff function of x and

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

or

$$\boxed{\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)}$$

$y = \sin 2x = 2 \sin x \cos x$

w/o chain rule

$y' = 2 \cos x \cos x + (-\sin x)(2 \sin x)$

$2 \cos^2 x - 2 \sin^2 x = 2(\cos^2 x - \sin^2 x) =$

$2 \cos 2x$

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with chain rule

$y = \sin 2x$

or

$y' = 2 \cos 2x$

$y = \sin \theta \quad \theta = 2x$

$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$

$(\cos \theta)(2) = 2 \cos 2x$

$$y = \sqrt{\sin^3(x^2+2x)}$$

$$y = \sqrt{w} \qquad w = u^3 \qquad u = \sin\theta \qquad \theta = x^2+2x$$

$$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{du} \cdot \frac{du}{d\theta} \cdot \frac{d\theta}{dx}$$

to be continued ....