22 Sept 2009

p. 129 133) $f(x) = x|x| = \begin{cases} -x^2 & \text{if } x<0 \\ 0 & \text{if } x=0 \\ x^2 & \text{if } x>0 \end{cases}$

$$f'(x) = \begin{cases} -2x & \text{if } x<0 \\ ? & \text{if } x=0 \\ 2x & \text{if } x>0 \end{cases}$$

$f'_{+}(0) = 2(0) = 0 \qquad f'_{(-)}(0) = -2(0) = 0$

$\therefore f'(0) = 0 \qquad f'(x) = \begin{cases} -2x & \text{if } x<0 \\ 0 & \text{if } x=0 \\ 2x & \text{if } x>0 \end{cases}$

135) $f(x) = ax^2 + bx + c$

$m = 10$ at $(2, 7)$

x int $(1, 0)$

$f'(x) = 2ax + b$

$f'(2) = \underline{4a + b = 10}$

$y - 7 = 10(x-2)$

$\underline{y = 10x - 13}$

$\cancel{y - 0 = 10(x-1)}$

$\underline{\cancel{y = 10x - 10}}$

$f(1) = a + b + c = 0$

$f(2) = 4a + 2b + c = 7$

$3a + b = 7$

$\underline{-4a - b = -10}$

$-a = -3$

$a = 3$

$b = -2$

$c = -1$

1 2c) ~~$f(x) = x^n$~~ $f(x) = \frac{1}{x} = x^{-1}$

$$y' = -1x^{-2} = \frac{-1}{x^2}$$

$$y'' = (-1)(-2)x^{-3} = \frac{2}{x^3}$$

$$y''' = (-1)(-2)(-3)x^{-4} = \frac{-6}{x^4}$$

$$f^n(x) = \frac{(-1)^n n!}{x^{n+1}}$$

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$f(x) = x^4 - 3x^2 + 2$

$(x^2-1)(x^2-2) = 0$

$x^2 = 1 \quad x^2 = 2$

$\boxed{\text{roots } 1, -1, \sqrt{2}, -\sqrt{2}}$

$f'(x) = 4x^3 - 6x$

$f'(1) = 4 - 6 = -2$

$f(1) = 1 - 3 + 2 = 0$

$\boxed{y = -2(x-1)}$

// to $y = -2x + 4$

$4x^2 + 4x - 2 = 0$

$2(2x^2 + 2x - 1) = 0$

$x = \frac{-2 \pm \sqrt{4+8}}{4}$

$4x^3 - 6x = -2$

$4x^3 - 6x + 2 = 0$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -6 & 2 \\ & & 4 & 4 & -2 \\ \hline & 4 & 4 & -2 & \end{array}$$

$\to \frac{-2 \pm 2\sqrt{3}}{4}$

$\boxed{\frac{-1 \pm \sqrt{3}}{2}, 1}$

Continued from 21 Sept

$$y = \sqrt[?]{\sin^3(x^2+2x)}$$

$y = \sqrt{w}$ $w = u^3$ $u = \sin\theta$ $\theta = x^2+2x$

$$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{du} \cdot \frac{du}{d\theta} \cdot \frac{d\theta}{dx}$$

$$\frac{1}{2\sqrt{w}} \cdot 3u^2 \cos\theta \, (2x+2)$$

$$\frac{dy}{dx} = \frac{3(\sin(x^2+2x))^2 \cos(x^2+2x)(2x+2)}{2\sqrt{(\sin(x^2+2x))^3}}$$

$$\frac{3(x+1)\sin^2(x^2+2x)\cos(x^2+2x)}{2\sqrt{\sin^3(x^2+2x)}}$$

$$y = \sqrt{\sin^3(x^2+2x)}$$

$$y' = \frac{3(\sin^2(x^2+2x))(\cos(x^2+2x))(2x+2)}{2\sqrt{\sin^3(x^2+2x)}}$$

$$y = f(g(x)) \quad \text{then} \quad y' = f'(g(x))\,g'(x)$$

$$h(x) = f(g(x))$$

$$\cancel{h'(x) = \lim_{\Delta x \to 0} \frac{f(g(x+\Delta x)) - f(g(x))}{\Delta x}}$$

$$h'(c) = \lim_{x \to c} \frac{f(g(x)) - f(g(c))}{x - c}$$

$$= \lim_{x \to c} \left[\frac{f(g(x)) - f(g(c))}{x - c} \cdot \frac{g(x) - g(c)}{g(x) - g(c)} \right]$$

$$= \lim_{x \to c} \frac{f(g(x)) - f(g(c))}{g(x) - g(c)} \cdot \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$$

let $g(x) = u$, $g(c) = b$

$$\lim_{u \to b} \frac{f(u) - f(b)}{u - b} \cdot \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$$

$$f'(b)\,g'(c)$$

$$f'(g(c))\,g'(c)$$

$\therefore$ any IF $c = x$ $\quad f'(g(x))\,g'(x)$