5 Oct 2009

54) cont on [a,b] and diff on (a,b)

$f(a) = f(b)$ $\Rightarrow$ $f'(c) = 0$

		int	c
a)	$g(x) = f(x) + k$	$[a,b]$	c
		$[a+k, b+k]$	$c+k$
b)	$g(x) = f(x-k)$	$[\frac{a}{k}, \frac{b}{k}]$	$\frac{c}{k}$
c)	$g(x) = f(kx)$		

$x-k=a$
$x-k=b$
$x-k=c$

$kx=a$ $kx=b$

6a) Prove $f(x) = 2x - 2 - \cos x$

f is cont and diffe for all x

Suppose f has more than 1 root

then there exist some a such that

$f(a)=0$ but some b such that $f(b)=0$

$\therefore$ Rolle's thm applies

but $f'(x) = 2 + \sin x \neq 0$ — $\therefore$ my assumption of more than 1 root is false

80) $f(x) = x^2 \qquad g(x) = -x^3 + x^2 + 3x + 2$

$f(-1) = 1 \qquad g(-1) = 1 + 1 - 3 + 2 = 1 \qquad f(-1) = g(-1)$

$f(2) = 4 \qquad g(2) = -8 + 4 + 6 + 2 = 4 \qquad f(2) = g(2)$

$f'(c_1) = \dfrac{f(2) - f(-1)}{2 - -1} \qquad g'(x) = -3x^2 + 2x + 3$

$f'(x) = 2x \qquad g'(c_2) = \dfrac{g(2) - g(-1)}{2 - -1}$

$2c_1 = \dfrac{4-1}{2+1} = 1 \qquad -3c_2^2 + 2c_2 + 3 = 1$

$c_1 = \frac{1}{2} \qquad 0 = 3c_2^2 - 2c_2 - 2$

$$c_2 = \frac{2 \pm \sqrt{4 + 24}}{6}$$

$$\frac{2 \pm 2\sqrt{7}}{6}$$

Let $h(x) = f(x) - g(x) \qquad 2x = -3x^2 + 2x + 3$

$h(-1) = 0 \qquad h(2) = 0 \qquad 3x^2 - 3 = 0$

$h'(c) = f'(c) - g'(c) = 0 \qquad 3(x^2 - 1) = 0$

$\boxed{x = 1} \quad \cancel{-1}$

$f'(c) = g'(c) \qquad c = 1$

42) $f(x) = \frac{x+1}{x} = 1 + \frac{1}{x}$ $\quad [1/2, 2]$

$f'(x) = -1/x^2$ $\quad -\frac{1}{c^2} = \frac{(\cancel{1}+1/2) - (\cancel{1}+2)}{2-1/2}$

73) $f(x) = x^{2n+1} + ax + b$

$f'(x) = (2n+1)x^{2n} + a$

$-\frac{1}{c^2} = \frac{-3/2}{3/2} = -1$

$c^2 = 1$

$\boxed{c = 1}$ $\cancel{-1}$

MVT $\quad$ f is cont on $[a,b]$ and diff on (a,b)

then there exists some

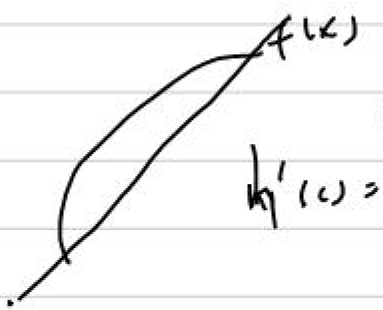
$c \in (a,b)$

such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Let h(x) = f(x) - $\cancel{\left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]}$

Let $h(x) = f(x) - \left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]$



Since $h(a) = h(b)$

$y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$

$h'(c) = 0$ $\quad h'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Increasing if for any $x_1 < x_2$ $f(x_1) < f(x_2)$

Decreasing if for any $x_1 < x_2$ $f(x_1) > f(x_2)$

Constant if for any $x_1 < x_2$ $f(x_1) = f(x_2)$

Thm for all $x \in [a, b]$

if $f'(x) > 0$ then f is increasing

if $f'(x) < 0$ then f is decreasing

if $f'(x) = 0$ then f is constant

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if $\underline{f'(x) > 0}$ by MVT

$x_1 < x_2$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c)$$

$$f(x_2) - f(x_1) > 0$$

$$\therefore \; f(x_2) > f(x_1)$$

1st derivative test for ~~rel~~ Relative Max and Min

1st Derivative Test

if f is cont on $[a,b]$ containing c and $(c, f(c))$ is a critical pt

if $f'(x)$ changes from negative to positive at $x=c$, then $(c, f(c))$ is a relative minimum.

if $f'(x)$ changes from positive to negative at $x=c$, then $(c, f(c))$ is a relative maximum.

$$f(x) = x^3 - \frac{3}{2}x^2 = \frac{1}{2}x^2(2x-3)$$

$$f'(x) = 3x^2 - 3x$$

$$\cancel{3x(x^2-3)}$$

$$3x(x-1)$$

x		0		1	
3	+ +		+ +		+ + +
x	− −	0	+ +		+ +
$x-1$	− −		− −	0	+ +
f'	+ +	0	− −	0	+ + +