15 Oct 2009

$V = \pi r^2 h$ V is constant

$S(r,h) = 2\pi r h + 2\pi r^2$

$h = \frac{V}{\pi r^2}$

$S(r) = 2\pi r\left(\frac{V}{\pi r^2}\right) + 2\pi r^2$

$S(r) = \frac{2V}{r} + 2\pi r^2$

$S'(r) = \frac{-2V}{r^2} + 4\pi r$

$S''(r) = \frac{4V}{r^3} + 4\pi > 0$

for any critical $r > 0$

∴ Any critical $r > 0$ will yield a min

$S'(r) = 0$

$4\pi r = \frac{2V}{r^2}$

$r^3 = \frac{2V}{4\pi} = \frac{V}{2\pi}$

$r = \sqrt[3]{\frac{V}{2\pi}}$

$h = \frac{V}{\pi\left(\frac{V}{2\pi}\right)^{2/3}}$

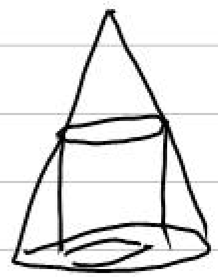
$h = \frac{4^{1/3} V^{1/3}}{\pi^{1/3}} = \sqrt[3]{\frac{4V}{\pi}}$

$\frac{h}{r} = \frac{\sqrt[3]{\frac{4V}{\pi}}}{\sqrt[3]{\frac{V}{2\pi}}}$

$\sqrt[3]{\frac{4V}{\pi}} \cdot \sqrt[3]{\frac{2\pi}{V}}$

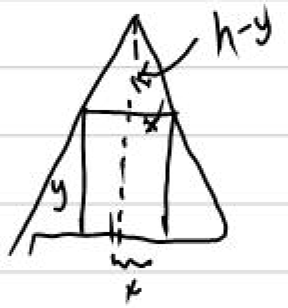
$\sqrt[3]{8} = 2$

$h = 2r$



Cone of a fixed r and h

what is largest cylinder that can be put inside?



$$V = \pi x^2 y$$

$$\frac{h-y}{x} = \frac{h}{r}$$

$$r(h-y) = hx$$
$$rh - ry = hx$$
$$rh - hx = ry$$
$$\frac{rh - hx}{r} = y$$

$$V(x) = \pi x^2 \left(\frac{rh - hx}{r}\right) \qquad [0, r]$$

$$V(x) = h\pi x^2 - \frac{\pi h}{r} x^3$$

$$V'(x) = 2\pi h x - \frac{3\pi h}{r} x^2 = 0$$

$$2\pi r h x - 3\pi h x^2 = 0$$

$$\pi h x(2r - 3x) = 0$$

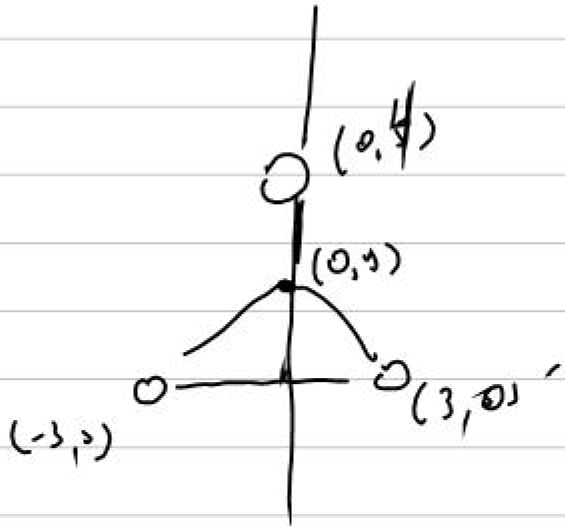
$$x = 0 \text{ or } x = \frac{2r}{3}$$

$$h\pi\left(\frac{4r^2}{9}\right) - \frac{\pi h}{r}\left(\frac{8r^3}{27}\right)$$

$$\frac{4hr^2\pi}{27}$$

$$V(0) = 0$$
$$V(r) = 0$$
$$V\left(\frac{2r}{3}\right) = \frac{4h^2\pi}{27}$$

is max is where ...



~~$D(y) = 4 - y + 2\sqrt{9+y^2}$~~

$$D(y) = 4 - y + 2\sqrt{9+y^2}$$

$y \in [0,4]$ $\quad D'(y) = -1 + \frac{2y}{\sqrt{9+y^2}} = 0$

$$\frac{2y}{\sqrt{9+y^2}} = 1$$

$4y^2 = 9 + y^2$

$3y^2 = 9$

$y^2 = 3$

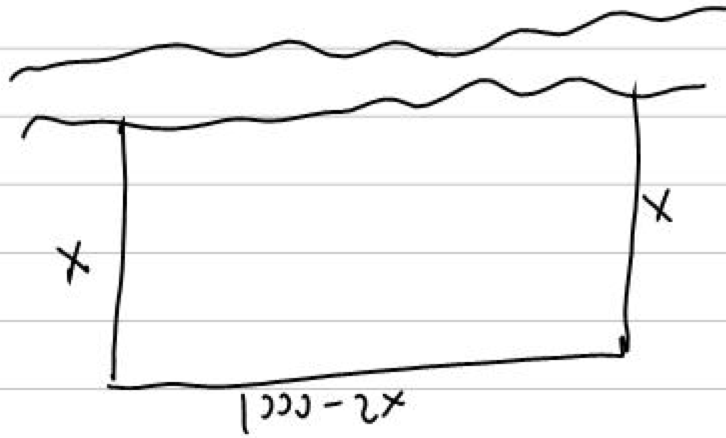
$y = \sqrt{3}$

$y(0) = 10$

$y(\sqrt{3}) = 4 - \sqrt{3} + 4\sqrt{3} = 4 + 3\sqrt{3}$

$y(4) = 10$

∴ min occurs when plant is at $(0, \sqrt{3})$ by closed interval test



1000 linear yds of fence

$A(x) = x(1000-2x) = 1000x - 2x^2$ $[0, 500]$

$A'(x) = 1000 - 4x$

critical $x = 250$

$A(0) = 0$

$A(500) = 0$

$A(250) = 125{,}000$

$(250)(500)$

build fence 250 by 500 by 250