Financial prob,

Marginal = Derivative
demand function = p ~ similar to price
revenue = $R = px$ = price (number of shirts)
Cost = $C(x)$
Profit = $P(x) = R(x) - C(x)$

Can sell 100 Tshirts if charge \$10 — assume demand function is linear
75 " " \$15

Cost = \$50 + \$6 per shirt

$m = -1/5$ — $(100, 10)$
$(75, 15)$

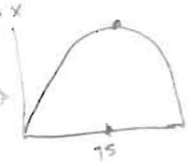
$p = -1/5(x - 100) + 10$

$p = \frac{-1}{5}x + 30$ — price, no one will buy any

Y int = demand if were free

$R(x) = -1/5\, x^2 + 30x$
$= -1/5\, x(x - 150)$

75 shirts = max



price = \$18
plug into price function

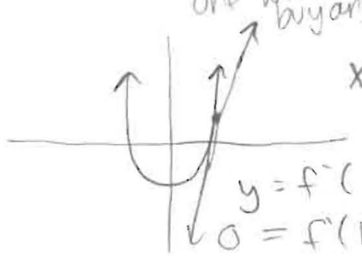
$C(x) = 6x + 50$

$P(x) = R(x) - C(x) = -1/5x^2 + 24x - 50$ $\quad P'(x) = -\frac{2}{5}x + 24$ $\quad 24 = \frac{2}{5}x$

$P''(x) = -\frac{2}{5} < 0$ $\quad x = 60$
∴ any critical point will yield max

Newton's Method

$x^2 - 2$



step ① guess at $\sqrt{2}$

$x_0 = 1.5$

② find an equation w/ tangent line through $f(x_0)$

$y = f'(x_0)(x - x_0) + f(x_0)$

③ solve for x intercept

$y = f'(1.5)(x - 1.5) + f(1.5)$

④ use that point as next guess

$0 = f'(1.5)(x - 1.5) + f(1.5)$

$1.5 - \frac{f(1.5)}{f'(1.5)} = x$

⑤ work until x values are very close

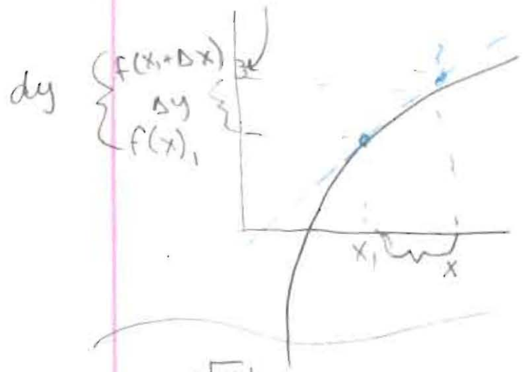
generic: $x_0 - \frac{f(x_0)}{f'(x_0)} = x_1$

* Know how to do a few steps

- Differential of x, called $d(x)$ is any nonzero number
- Differential y, called dy is $dy = f'(x)\,dx$ $\quad \frac{dy}{dx} = f'(x)$

ex. $f(x) = x^2 - 5x + 2$

$dy = (2x - 5)\,dx$



$\Delta x = dx$

$y - f(x_1) = f'(x_1)(x - x_1)$

$y = f(x_1) + f'(x_1)(\Delta x)$

$y = f(x_1) + dy$

$dy \approx \Delta y$

$dy = f'(x)dx$

$\sqrt{51}$

$f(x) = \sqrt{x}$

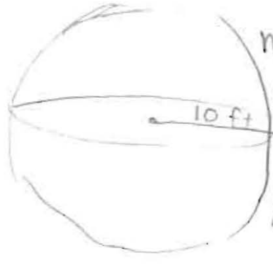
$x = 49$

$dy = \left(\frac{1}{2\sqrt{x}}\right)(dx)$

$\Delta x = dx = 2$

$\sqrt{51} = f(51) = f(49) + \Delta y \approx f(49) + dy$

$\sqrt{51} \approx 7 + \frac{1}{2(7)}(2) = 7\frac{1}{7}$



need volume of 1 inch shell

$V = \frac{4}{3}\pi r^3$

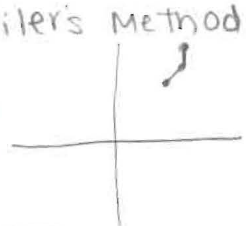
$V_{shell} = \frac{4}{3}\pi\left(10\frac{1}{12}\right)^3 - (4/3)\pi(10^3)$

$\Delta V \approx dv = 4\pi r^2 dr$

$= 4\pi(10^2)\left(\frac{1}{12}\right)$ underestimation

* do second derivative to find if over/under estimate
* if concave up, tangent is under line

Oiler's Method



$\frac{dy}{dx} = 5x - 2$ * will learn more later

$\sin 29°$

$f(x) = \sin x$

$dy = \cos x \, dx$

$x = 30°$

$dx = -\pi/180$ must be radian

$\sin 29° = \sin(30° + \Delta x) = \sin 30° + \Delta y \approx \sin 30° + dy$

$\frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-\pi}{180}\right)$

$dy = \cos x \, dx$

- graphically
test newton

- use differentials
to approximate word
Probs,

Inverse tan

$\tan^{-1}(x) = y$ $\quad -\frac{\pi}{2}, \frac{\pi}{2}$

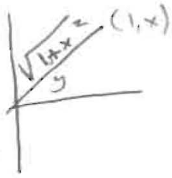
$\tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

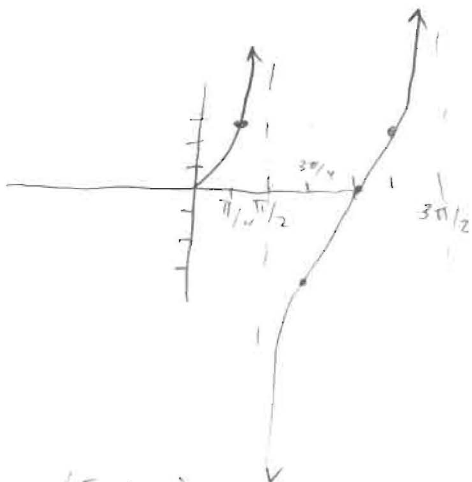
$\sec y = \frac{\sqrt{1+x^2}}{1}$



abscissa = x value
ordinate = y value

Newtons Method

$y = \tan x - x$



* if use a horizontal tangent, wont cross x axis

Des Cartes Rule of Signs (Extra)

$f(x) = 5x^4 - 3x^3 - 2x^2 + 5x - 7$

$+ \quad - \quad - \quad + \quad -$

$f(-x) + \quad + \quad - \quad - \quad -$

P	3	1
N	1	1
i	0	2

$\Delta V = \frac{4}{3}\pi(1.01)^3 - \frac{4}{3}\pi(1)^3$

$\frac{4}{3}\pi(.99)^3 - \frac{4}{3}\pi(1)^3$

Relative Error = $\frac{\Delta V}{V}$

$\frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} \approx .03 = 3\%$

- if have .01 deviation, V will have 3% error
- to find over/under approximation, take second derivative of $y = \frac{4}{3}\pi r^3$