Proof: $\ln(ab) = \ln a + \ln b$ $a > 0$ $b > 0$

$f(x) = \ln(ax)$ $g(x) = \ln a + \ln x$

$f'(x)\ \frac{1}{ax}(a)$ $g'(x) = 0 + \frac{1}{x}$

$\frac{1}{x}$ $\frac{1}{x}$

$\therefore$ functions differ by a constant

$\ln(ax) = \ln a + \ln x + c$

$\ln(a \cdot 1) = \ln a + \ln 1 + c$

$\therefore\ c = 0$

Proof. $\log_a m = x$ $\log_a N = y$

$a^x = m$ $a^y = N$

$NN = a^x a^y$

$mN = a^{x+1}$

$\log_a(mn) = x + y$

$\log_a(mn) + \log_a m + \log_a N$

Rule: $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ $a > b,\ b > 0$

$\frac{d}{dx} \ln|x|$

if $x > 0$

$\frac{d}{ax} \ln x \left(\frac{1}{x}\right)$

$\frac{1}{x}$

if $x < 0$

$\frac{d}{dx} \ln(-x)$

$\frac{1}{-x}(-1)$

$\frac{1}{x}$

1. $\int \frac{1}{x} dt = \ln|x| + c$

Rules:

$\ln|ab| = \ln|a| + \ln|b|$

$\ln\left|\frac{a}{b}\right| = \ln|a| - \ln|b|$

$\ln|a^n| = N \ln|a|$

Logarithmic Differentiation

$y = \frac{(x+5)^4(x-2)^3}{(x+5)^7}$ & you want to take the derivative

$|y| = \frac{|x+3|^4|x-2|^3}{|x+5|^7}$

$\ln|y| = \frac{\ln|x+3|^4(x-2|^3}{|x+5|^7}$

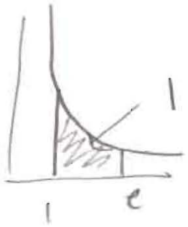
$\ln|y| = 4\ln|x+3| + 3\ln|x-2| - 7\ln|x+5|$

$\frac{1}{y}(D_xy) = \frac{4}{x+3} + \frac{3}{x-2} - \frac{7}{x+5}$

$\therefore D_xy = \frac{(x+3)^4(x-2)^3}{(x+5)^7}\left(\frac{4}{x+3} + \frac{3}{x-2} - \frac{7}{x+5}\right.$

$\int_1^e \frac{1}{t}\,at = 1$

$\ln x = 1$



$\frac{d}{dx}\ln|x| = \frac{1}{x}$

$\int \frac{1}{x}\,ax = \ln|x| + c$

$\int \frac{du}{u} = \ln|u| + c$

$\int \tan x\,dx$ $u = \cos x$

$-\int -\frac{\sin x}{\cos x} = -\int \frac{du}{u}$ $du = -\sin x\,ax$

$= -\ln|u| + c$

$= -\ln|\cos x| + c$

$= \ln|\cos x^{-1}| + c$

$= \ln\left|\frac{1}{\cos x} + c\right.$

$= \ln|\sec x| + c$

$\int \frac{1}{x}\,dx = \ln|x| + c$

$\int \tan x\,dx = \begin{cases} -\ln|\cos x| + c \\ \ln|\sec x| + c \end{cases}$ ← preferred

$\int \cot x\,dx = \begin{cases} \ln|\sin x| + c \\ -\ln|\csc x| + c \end{cases}$

$\int \cos x\,dx = \sin x$

$\int \sec x\,ax = \begin{cases} \ln|\sec x + \tan x| + c \\ -\ln|\sec x - \tan x| + c \end{cases}$

$\int \csc\,dx = \begin{cases} -\ln|\csc x + \cot x| + c \\ \ln|\csc x - \cot x| + c \end{cases}$

$\int \cot x\,dx$

$\int \frac{\cos x}{\sin x}\,ax$

$u = \sin x$

$du = \cos x\,dx$

$\int \frac{du}{u} = \ln|u| + c$

$\ln|\sin x| + c$

$\ln|(\csc x)^{-1}| + c$

$-\ln|\csc x| + c$

$\int \sec x\,dx$ * no u substitution with variables!

$\int \sec x \frac{(\sec x + \tan x)}{(\sec x + \tan x)}\,du$

$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$

$u = \sec x + \tan x$

$du = (\sec x + \sec^2 x)$

$\int \sec x\,dx = \begin{cases} \ln|\sec x + \tan x| + c \\ -\ln|\sec x - \tan x| + c \end{cases}$

$\int \csc x\,dx = \begin{cases} -\ln|\csc x + \cot x| + c \\ \ln|\csc - \cot x| + c \end{cases}$

$\int \frac{\csc x}{1}\left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)dx$

$-\int \frac{-\csc^2 x + \csc x \cot x}{\csc x + \cot x}$

$-\ln|\csc x + \cot x| + c$

$-\ln\left|\frac{1+\cos x}{\sin x}\right| + c$

* -When have rational expression + degree in numerator ≥ degree of denominator, divide out first *

$\int \frac{x}{x^2+4}\,dx$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}\int 2x / x^2+4$

good for FR → $\frac{1}{2}\ln|x^2+4| + c$

$\frac{1}{2}\ln(x^2+4) + c$

expect in MC → $\ln\sqrt{x^2+4} + c$

$\cancel{\int \frac{dx}{x^2+4}}$

Can't do yet

$= \frac{1}{2}\text{Arctan}\frac{x}{2} + c$

$\int \frac{x^3}{x^2+4}\,dx$

$$\begin{array}{r} x - \frac{4x}{x^2+4} \\ x^2+4 \overline{)\, x^3 \qquad} \\ \underline{x^3 + 4x} \\ -4x \end{array}$$

$\int \left(x - \frac{4x}{x^2+4}\right)dx$

$\frac{x^2}{2} - 2\ln(x^2+4) + c$

$\int \frac{x-1}{x+3}\,dx$

$$\begin{array}{r} 1 - \frac{4}{x+3} \\ x+3 \overline{)\, x - 1} \\ \underline{x + 3} \\ -4 \end{array}$$

$\int \left(1 - \frac{4}{x+3}\right)dx$

$x - 4\ln|x+3| + c$ ← dont forget abs| |

$x - \ln|3x+3| + c$

$= x - \ln 3 + \ln(x+1) + c$

$= x + \ln|x+1| + c$ ← for MC are equal because the c can change

$\int \frac{\overset{u}{\ln x}}{x}\overset{du}{dx}$

$u = \ln x$

$du = \frac{1}{x}$

$\int u\,du$

$\frac{u^2}{2} + C$

$\frac{(\ln x)^2}{2} + C$

$\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{1}{x}$

$\int \frac{du}{u} = \ln|u| + c$

$\ln|\ln x| + C$

$\int \cancel{\ln} x\,dx$

Cant do yet

$\int_0^{\pi/4} \sqrt{\tan^2 x + 1}\quad dx$

$\int_0^{\pi/4} \sqrt{\sec^2 x}$

$\int_0^{\pi/4} \sec x$ ← use absolute value usually, but here sec is pos from $[0, \pi/4]$

$\ln(\sqrt{2}+1) - \ln|1+0|$