$$\int \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}$

$\int u\,du$

$\frac{u^2}{2} + C$

$\frac{(\ln x)^2}{2} + C$

$$\int \frac{dx}{x \ln x}$$

$u = \ln x$

$du = \frac{1}{x}$

$\int \frac{du}{u} = \ln|u| + c$

$\ln|\ln x| + C$

$\int \cancel{\ln} x\,dx$

Can't do yet

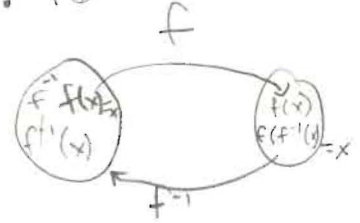
$$\int_0^{\pi/4} \sqrt{\tan^2 x + 1}\;dx$$

$\int_0^{\pi/4} \sqrt{\sec^2 x}$

$\int_0^{\pi/4} \sec x$ ← use absolute value usually but here sec is pos from $[0, \pi/4]$

$\ln(\sqrt{2}+1) - \ln|1+0|$

Nov. 16



$f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = x$

$y = \frac{3x-2}{5x+7}$ $\quad D_f: (-\infty, -7/5) \cup (-7/5, \infty) \cup (\frac{3}{5}, \infty)$

$R_f: (-\infty, \frac{3}{5}) \cup (3/5, \infty)$

$\frac{3y-2}{5y+7} = x$

$3y - 2 = 5xy + 7x$

$3y - 5xy = 7x + 2$

$y(3 - 5x) = 7x + 2$

$y = \frac{7x+2}{3-5x}$

* Domain of inverse = range of original

if $g(x) = f^{-1}(x)$

$$f(g(x)) = x$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$f'(g(x))\, g'(x) = 1$$

$$\boxed{g'(b) = \frac{1}{f'(a)}}$$

Only works for derivatives at points

$$\boxed{\begin{array}{l} f(a) = b \\ g(b) = a \end{array}}$$

$f(x) = y = 8x^5 + x^3 + 5x - 7$

$y' = 40x^4 + 3x^2 + 5$

* Derivative always positive, so always increasing (no turns)

$$(f^{-1})'(7) = \frac{1}{f'(1)} = \frac{1}{48}$$

always guess 0, 1, -1, 2 & plug $f(x)$

Monotonic - non increasing or non decreasing (could be constant)

Strictly monotonic - always increasing or decreasing

* if a function is strictly monotonic, there is an inverse

$y = \ln x$

$f^{-1}(x) = e^x$ named after Euler