if $g(x) = f^{-1}(x)$

$f(g(x)) = x$

$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$

$f'(g(x))\,g'(x) = 1$

$\boxed{g'(b) = \frac{1}{f'(a)}}$ ← only works for derivatives at points

$\boxed{\begin{matrix} f(a) = b \\ g(b) = a \end{matrix}}$

$f(x)$: $y = 8x^5 + x^3 + 5x - 7$

$y' = 40x^4 + 3x^2 + 5$

* Derivative always positive, so always increasing (no turns)

$f^{-1\prime}(7) = \frac{1}{f'(1)} = \frac{1}{48}$ always guess 0, 1, -1, 2 + plug $f(x)$

Monotonic - non increasing or non ~~de~~creasing (could be constant)

Strictly monotonic - always increasing or decreasing

* if a function is strictly monotonic, there is an inverse

$y = \ln x$

$f^{-1}(x) = e^x$ named after Euler

Nov. 17

$f(x) = \ln x$

$f^{-1}(x) = g(x) = e^x$

$\ln x = y \Leftrightarrow e^y = x$

$e^x = y \Leftrightarrow \ln y = x$

$e^{\ln x} = x$ → $e^{\ln(\text{ugly})} = \text{ugly}$

$\ln(e^x) = x$

$y = x^x = e^{\ln x^x} = e^{x\ln x}$

$D_x x^x = e^{x\ln x}(1\ln x + \frac{1}{x}x)$

$D_x x^x = x^x(\ln x + 1)$

$D_x e^x = e^x$

$y = e^x$

$\ln y = x$

$\frac{1}{y}\frac{dy}{dx} = 1$

$\frac{dy}{dx} = y$

$\frac{dy}{dx} = e^x$

$\therefore \frac{d}{dx}e^u = e^u\frac{du}{dx}$

$D_x y = y$

$\frac{dy}{dx} = ky$

$dy = ky\,dx$

$\int \frac{dy}{y} = \int k\,dx$

$\ln|y| = kx + C_1$

$|y| = e^{kx + C_1}$

$y = \pm e^{kx + C_1}$

$y = \pm e^x e^{C_1}$ constant

$y = \pm Ce^x$

$\therefore$ any scaler multiple works for $D_x e^x = e^x$

$\frac{Dy}{Dx} = ky$

$\therefore$ if rate of change is proportional to amount present, go from $\frac{dy}{dx} = ky$ to $y = Ce^{kx}$

$y = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{e^{\frac{x^2}{2}}}$

D: $(-\infty, \infty)$

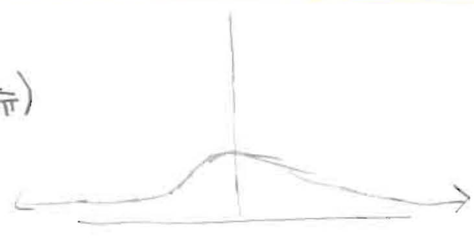
R: $(0, \frac{1}{\sqrt{2\pi}})$

$y' = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}(-x)$

y' + + + | − − − (0)

inc dec

max at 0



$y'' = x^2\left(\frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}\right) - \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}$

$\frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}(x^2 - 1)$

y'' + + + (−1) − − − (1) + + +

up down up

$(\pm 1, \frac{1}{\sqrt{2\pi e}})$

$e^a e^b = e^{a+b}$ $a>0$ $b>0$

$\ln(ab) = \ln a + \ln b$

$\ln(e^a e^b) = a+b$

$\ln e^a + \ln e^b$

$a+b$

$\int e^x = e^x + C$

$\int e^{x^2} dx$

$\log_a x = y$

$a^y = x$

$\log_{10} x = \log x$

$\log_e x = \ln x$

$a^y = x$

$\log_b a^y = \log_b x$

$y \log_b a = \log_b x$

$y = \dfrac{\log_b x}{\log_b a} = \dfrac{\ln x}{\ln a}$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} = \log_a x = \frac{1}{x \ln a}$

$\frac{d}{dx} e^x = e^x$ $\frac{d}{dx} a^x = a^x \ln a$ $y = a^x$

$y = a^x = e^{\ln a^x} = e^{x \ln a}$

$y' = e^{x \ln a}(\ln a) = a^x \ln a$ or $a^u \ln a \frac{du}{dx}$

$D_x 2^{x^2-3x} = 2^{x^2-3x}(\ln 2)(2x-3)$ or

$D_x e^{\ln(2^{x^2-3x})}$

$D_x e^{(x^2-3x)(\ln 2)}$

$= 2^{x^2-3x}(\ln 2)(2x-3$

$y = a^x$

$\ln y = \ln a^x$

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$D_x y = y \ln a$

$D_x a^x = a^x \ln a$

$D_x a^{(u(x))} = a^{u(x)} \ln a \,(u'(x))$

$\int e^x dx = e^x + C$

$\int a^x dx$

$= \int e^{\ln a^x}$

$\int e^{x \ln a}$

$u = x\ln a$

$du = (\ln a)dx$

$\frac{1}{\ln a} \int \ln a \, e^{x \ln a} dx$

$= \frac{1}{\ln a} \int e^u du = \frac{1}{\ln a} e^u + C$

$= \frac{1}{\ln a} e^{x \ln a} + C$

$= \frac{1}{\ln a} e^{\ln a^x} + C$

$\frac{1}{\ln} a^x + C$

$= \frac{a^x}{\ln a} + C$

$\boxed{\int a^x dx = \frac{a^x}{\ln a} + C}$

$A = P(1+\frac{r}{n})^{nt}$

\$1000

After 4 years

Bank SL 3.5%
compound yearly
\$1,147.52

Bank B 3.25%
compound daily
\$1,138.82

Bank of Dallas
3.5%
compound daily
\$1,150.26

Bank of MO city
continuous compount
$\lim_{N\to\infty} P(1+\frac{r}{N})^{t} = Pe^{rt}$
\$1,150.27

Nov. 19

$\lim_{x\to\infty}(1+\frac{1}{x})^{x} = e$ - known

$\lim_{N\to\infty} P(1+\frac{r}{n})^{nt}$

let $x = \frac{n}{r}$

$P\lim_{N\to\infty}(1+\frac{r}{n})^{nt}$

$P\lim_{x\to\infty}(1+\frac{1}{x})^{rxt}$

$P\left[\lim_{x\to\infty}(1+\frac{1}{x})^{x}\right]^{rt}$

Pe^{rt}

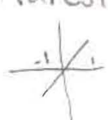
Inverse Trig

$y = \arcsin x = \sin^{-1}x$

D: $-1 \le x \le x$

R: $-\pi/2 \le x \le \frac{\pi}{2}$

$\sin(\arcsin x) = x$

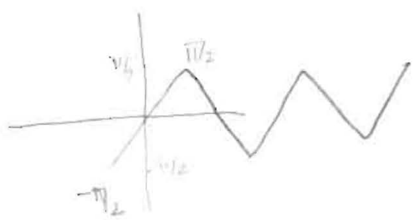


* true between -1 and 1

$\arcsin(\sin x) = x$

$\arcsin(\sin 2\pi/3) = \pi/3$

not $2\pi/3$



$\frac{d}{dx} \sin(\arcsin x) \quad \frac{d}{dx} x$

$\cos(\arcsin \quad) D_x \arcsin x = 1$

$D_x \arcsin x = \frac{1}{\cos(\arccos x)}$

$D_x \arcsin x = \frac{1}{\sqrt{1-x^2}}$

$D_x \arccos x = \frac{-1}{1-x^2}$

* chain rule applies

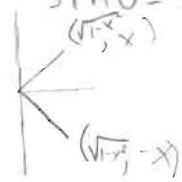
ex. $\frac{d}{dx} \arcsin e^x$

$= \frac{1 \, e^x}{\sqrt{1-e^{2x}}}$

$\cos(\arcsin x)$

$\arcsin x = \theta$

$\sin\theta = x$



$a^2 + x^2 = 1$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\cos\theta = \sqrt{1-x^2}$

$\arccos x = y$

$\cos y = x$

$\arcsin x = -\arccos x + C$

$\frac{d}{dx} \cos y = \frac{d}{dx} x$

$-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{-1}{\sin y}$