$$\int \frac{dx}{x\sqrt{x^2-a^2}}$$

$$\int \frac{\frac{a}{\;}dx}{a x\sqrt{(\frac{x}{a})^2-1}} \qquad u=\frac{x}{a} \quad du=\frac{1}{a}dx \quad x=au$$

$$\frac{1}{a}\int \frac{du}{u\sqrt{u^2-1}}$$

$$=\frac{1}{a}\operatorname{arcsec}|u|+C$$

$$=\frac{1}{a}\operatorname{arcsec}\frac{|x|}{a}+C$$

Chapter 6

Dec. 2 - Slope Fields

$\frac{dy}{dx} = x+y$

* must change direction + steepness

Euler's Method

$\frac{dy}{dx} = x+y \quad (0,1)$

$f(1) \approx$

$y-1 = 1(x-0)$

$y = x+1$

$f(1) \approx 2$

$f(2) \approx$

$y-1 = 1(x-1$

$y = x+1$

$f(2) \approx 3$

* very off on slope field

Cont ⇒

Integral Rules

$f\left(\frac{1}{2}\right) = \frac{3}{2}$

$\left(\frac{1}{2}, \frac{3}{2}\right)$

$m = 2$

$y = 2\left(x - \frac{1}{2}\right) + \frac{3}{2}$ $\therefore$ $y(1) = 5/2$

* use a line

- line at 1
- plug in $\frac{3}{2}$
- find line at $\frac{3}{2}$
- plug in 2

$y = \frac{7}{2}(x-1) + \frac{5}{2}$ at 1

$y\left(\frac{3}{2}\right) = \frac{7}{4} + \frac{5}{2} = \frac{17}{4}$ $\left(\frac{3}{2}, \frac{11}{4}\right)$ $m = \frac{23}{4}$

$\frac{dy}{dx} = \frac{x}{y}$ $(0,1)$

$y\,dy = x\,dx$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C_1$

$y^2 = x^2 + C_2$

$y = \pm\sqrt{x^2 + C_2}$ *know pos. b/c (0,1)

$y = \sqrt{x^2 + c}$

$1 = \sqrt{0^2 + c}$

$1 = c$

$y = \sqrt{x^2 + 1}$

- hyperbola

* pay attention to domains

$\frac{dy}{dx} = \frac{y}{x}$

$dy = \frac{y}{x}\,dx$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = \ln|x| + C_1$

$|y| = e^{\ln|x| + C_1} = e^{\ln|x|} e^{C_1}$

$y = c|x|$

$(2, 6)$

$6 = c|2|$

$3 = c$

$y = 3|x|$

Domain $y = 3x$ or $y = 3|x|, x > 0$

* can't go past point w/ domain restriction in derivative b/c we don't know what happens after

$\frac{dy}{dt} = ky$ — Rate of change is proportional to amount present

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + c_1$

$|y| = e^{kt+c_1} = e^{kt}e^{c_1}$

$y = Ce^{kt}$

Newton's Law of Cooling

$\frac{dT}{dt} = k(T-M)$ M- temp. of medium (constant

$\int \frac{dT}{T-M} = \int k\,dt$

$\ln|T-m| = kt + c_1$

$|T-m| = e^{kt+c} = e^{kt}e^{c_1}$

$T - m = Ce^{kt}$

$T = m + Ce^{kt}$

- room = 75°
- (T, time), (350, 0)

$T = 75 + 275e^{kt}$

- solve for k with (350,0)
- then

Dec 3

Logistic Curve *on test* (Walmart Curve)

$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ ← same → $\frac{dy}{dt} = ky(L-y)$

$= \frac{k}{L}y(L-y)$

$y = \frac{L}{1+be^{-kt}}$

theory

$\frac{1}{y(L-y)} = \frac{A}{y} + \frac{B}{L-y}$

$1 = A(L-y) + B(y)$

$y=0 \quad A = \frac{1}{L}$

$A = L \quad B = \frac{1}{L}$

$\int \frac{dy}{y(L-y)} = \int k\,dt$

$\int \left(\frac{\frac{1}{L}}{y} + \frac{\frac{1}{L}}{y}\right)dy =$

$= \int k\,dt$

$= \frac{1}{L}\left(\ln|y| - \ln|L-y|\right) = kt + c_1$

$= \ln\left|\frac{y}{L-y}\right| = Lkt + C_2$

$\left|\frac{y}{L-y}\right| = e^{Lkt} + e^{C_2}$

$\frac{y}{L-y} = C_3e^{Lkt}$

$\frac{y}{L-y} = \frac{1}{C_3e^{Lkt}} = C_4e^{-Lkt}$

$\frac{L}{y} = 1 + C_4e^{-Lkt}$

→

Integral Rules

$$\frac{y}{L} = \frac{1}{1+C_4 e^{-LKt}}$$

$$\boxed{y = \frac{L}{1+Ce^{-LKt}}}$$

* must start w/ $\frac{dy}{dt} = Ky(L-y)$

$\lim_{t \to \infty} y = L$

Curve

ex. $\frac{dy}{dt} = y\left(1-\frac{y}{40}\right)$ $(0, 8)$

$y = \frac{40}{1+be^{-1t}}$

$8 = \frac{40}{1+b}$

$b = 4$

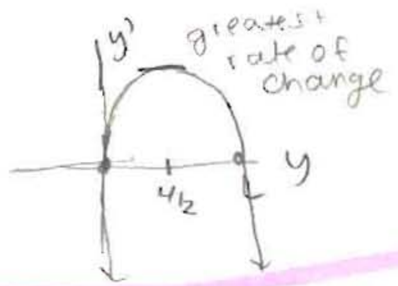
OR

$\frac{dy}{dt} = \frac{1}{40}y(40-y)$

$y = \frac{40}{1+Ce^{-t}}$

. . . etc.

$\frac{dy}{dt} = Ky(L-y)$



* grow fastest when $\frac{L}{2}$ on test

Cant seperate:

$(x^2+xy)\,dx\ L y^2\,dy = 0$