

40dek, 5 yrs 104 e|k, L=4,000

$$\frac{dy}{dt} = ky(4000-y)$$

* k is diffr.

$$y = \frac{L}{1+be^{-kt}} \quad \text{or } y = \frac{L}{1+ce^{-kt}}$$

$$y = \frac{L}{1+ce^{-kt}} \quad \leftarrow \quad y = \frac{L}{1+be^{-kt}}$$

$$40 = \frac{40,000}{1+c}$$

$$40 + 40c = 40,000$$

$$c = 99$$

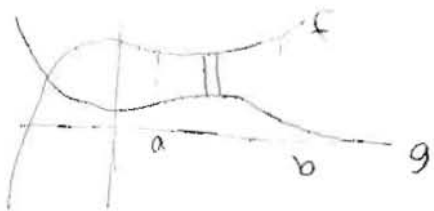
$$104 = \frac{4000}{1+99e^{-5k}}$$

$$e^{-5k} = \frac{4000-104}{(104)(99)}$$

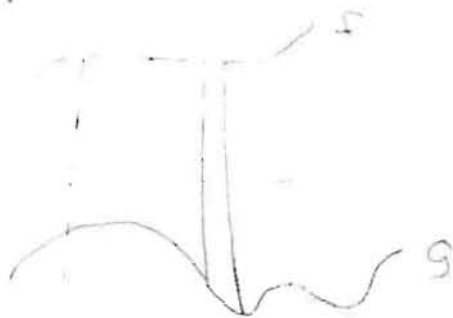
$$-5k = \ln\left(\frac{4000-104}{104(99)}\right)$$

$$k = \frac{\ln\left(\frac{4000-104}{104(99)}\right)}{-5}$$

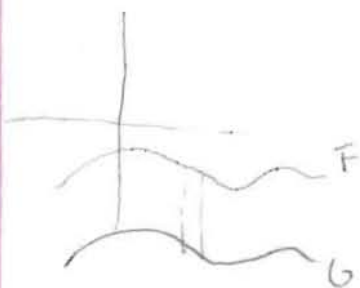
Chapter 7



$$f(x) - g(x)$$



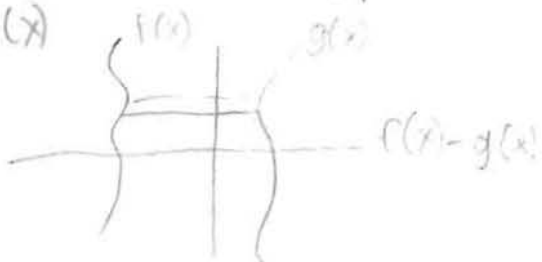
$$f(x) - g(x)$$



$$f(x) - g(x)$$

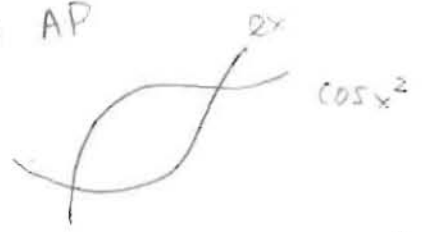
* if function in terms of y, subtract right \rightarrow left

* if function in terms of x, subtract top-bottom



$$\int_a^b f(x) - g(x) dx$$

On AP



$$y_1 = f$$

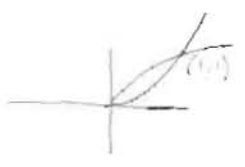
$$y_2 = g$$

$$fint (y_1, y_2, x, a, b)$$

$$\int_{.123}^{.567} \cos x^2 - e^{x^2} dx$$

$$y = \sqrt{x}$$

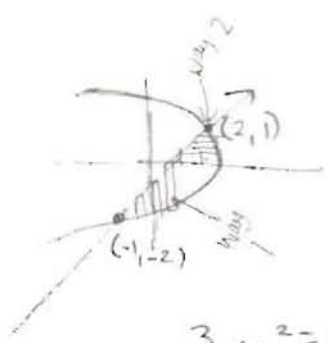
$$x = \sqrt{y}$$



12/19/09

$$x = 3 - y^2$$

$$x - y = 1$$



$$y = x - 1$$

$$y = \sqrt{3 - x}$$

$$y = -\sqrt{3 - x}$$

$$3 - y^2 = 1 + y$$

$$0 = y^2 + y - 2$$

$$0 = (y - 1)(y + 2)$$

$$(2, 1) (-1, -2)$$

$$\int_{-1}^2 (x-1) - (-\sqrt{3-x}) dx$$

$$\int_{-1}^2 (x-1) - (-\sqrt{3-x}) dx + \int_2^3 (\sqrt{3-x} - (-\sqrt{3-x})) dx$$

$$\left[\frac{x^2}{2} - x - \frac{2}{3} (3-x)^{3/2} \right]_{-1}^2 + -2 \left[\frac{2}{3} (3-x)^{3/2} \right]_2^3 - \int_2^3 2\sqrt{3-x} dx$$

$$\left(2 - 2 - \frac{2}{3} \right) - \left(\frac{1}{2} + 1 - \frac{16}{3} \right) - \frac{4}{3} (0-1) - 2 \left(\frac{2}{3} (3-x)^{3/2} \right)$$

$$6 - \frac{3}{2}$$

$$4 \frac{1}{2}$$

OR

$$\int_{-2}^1 [(3-y^2) - (y+1)] dy$$

$$\int_{-2}^1 (2 - y^2 - y) dy$$

$$= \left[2y - \frac{y^3}{3} - \frac{y^2}{2} \right]_{-2}^1$$

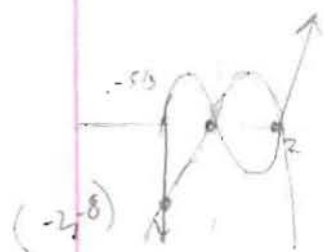
$$= \left(2 \cdot \frac{1}{3} - \frac{1}{2} - \frac{1}{2} \right) - \left(-4 - \frac{8}{3} - 2 \right)$$

$$8 - 3 - \frac{1}{2}$$

$$= 4 \frac{1}{2}$$

$$f(x) = 3x^2 - x^2 - 10x = x(3x^2 - x - 10) = x(3x+5)(x-2)$$

$$g(x) = -x^2 + 2x = -x(x-2)$$



$$x(3x+5)(x-2) = -x(x-2)$$

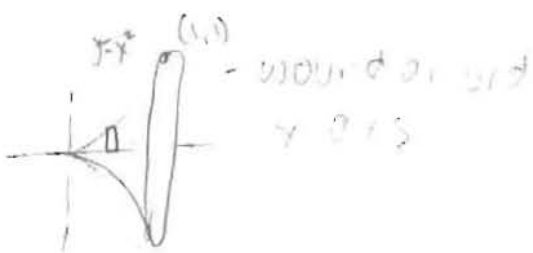
$$(3x+5) = -1$$

$$3x = -6$$

$$x = -2$$

$$\int_{-2}^0 [(3x^2 - x^2 - 10x) - (-x^2 + 2x)] dx + \int_0^2 [(-x^2 + 2x) - (3x^2 - x^2 - 10x)] dx$$

Disk Method



To find Volume:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \pi (f(c_i))^2 \Delta x_i$$

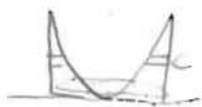
$$\pi \int_a^b (f(x))^2 dx$$

$$\pi \int_0^1 x^4 dx$$

$$\pi \left(\frac{x^5}{5} \right) \Big|_0^1$$

$$= \frac{\pi}{5}$$

Washer Method



washer

$$\pi R^2 h - \pi r^2 h$$

$$\pi h (R^2 - r^2)$$

Why Values

$$\pi \int_0^1 (1^2 - x^2) dy$$

$$\pi \int_0^1 (1-y) dy = \pi \left(y - \frac{y^2}{2} \right) \Big|_0^1$$

$$\pi \left(\frac{1}{2} \right) = \frac{\pi}{2}$$

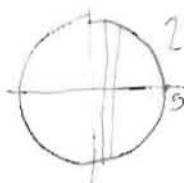
* whoever gets

differential wins

$$l^2 = R \quad x^2 = r^2 \quad h = dy$$

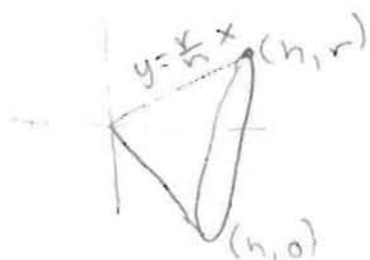
Dec. 10

$$x^2 + y^2 = 25$$



$$2 \int_0^5 (2y)^2 dx = 2 \int_0^5 4(25 - x^2) dx$$

Proving Volume of a Cone



$$\pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$\frac{\pi r^2}{h} \left[\frac{x^3}{3}\right]_0^h$$

$$\frac{r^2 \pi h^3}{3h^2} = \frac{r^2 \pi h}{3}$$

Sphere



$$y = \sqrt{r^2 - x^2}$$

$$2\pi \int_0^r (\sqrt{r^2 - x^2})^2 dx$$

$$2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r$$

$$2\pi \left(r^3 - \frac{r^3}{3} \right) = 0$$

$$\frac{4}{3} \pi r^3$$



Square Pyramid

$$\frac{-h}{b/2} = \frac{-2h}{b}$$

$$y = -\frac{2h}{b}x + h$$

same line

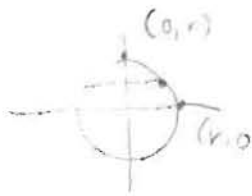
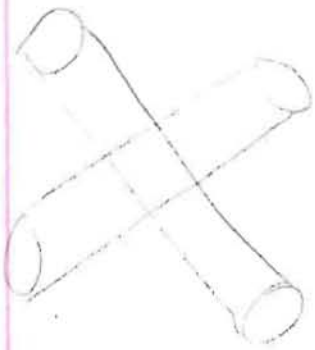
$$\int_0^h \left[\frac{b}{2h}(y-h) \right]^2 dy$$

$$\frac{b^2}{h^2} \left[\frac{(y-h)^3}{3} \right]_0^h$$

$$y-h = du$$

$$\frac{b^2}{3h} (0 - -h^3)$$

$$= \boxed{\frac{b^2 h}{3}}$$



$$2 \int_0^r (2x)^2 dy$$

$$8 \int_0^r (r^2 - y^2) dy = 8 \left(ry - \frac{y^3}{3} \right) \Big|_0^r$$

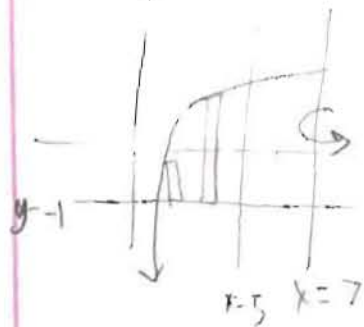
$$= 8 \left(\frac{2r^3}{3} - 0 \right)$$

$$= \frac{16r^3}{3}$$



$$V = 2\pi r h dx$$

$$2\pi \int x f(x) dx$$



Shell:

$$2\pi \int_{e^{-1}}^5 (7-x)(\ln x + 1) dx$$

$$\ln x = -1$$

$$x = e^{-1}$$

Washer:

$$\pi \int_{-1}^{\ln 5} (7-x)^2 - 2^2 dy$$

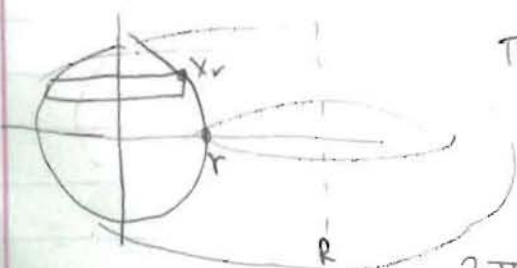
$$y = \ln x$$

$$x = e^y$$

$$\pi \int_{-1}^{\ln 5} (7 - e^y)^2 - 2^2 dy$$

* If write

$$\int_a^b (R-r)^2, \text{ no partial credit on AP}$$



$$\pi \int_0^r (R-x_L)^2 - (R-x_R)^2 dy$$

$$x^2 + y^2 = r^2$$

$$x = \pm \sqrt{r^2 - y^2}$$

$$2\pi \int_0^r (R + \sqrt{r^2 - y^2})^2 - (R - \sqrt{r^2 - y^2})^2 dy$$

$$2\pi \int_0^r (R^2 + 2R\sqrt{r^2 - y^2} + r^2 - y^2 - R^2 + 2R\sqrt{r^2 - y^2} - (r^2 - y^2)) dy$$

SEMICIRCLE

$$\begin{aligned} & 8\pi R \int_0^r \sqrt{r^2 - y^2} dy \\ & 8\pi R \left(\frac{1}{4} \pi r^2 \right) \\ & = 2\pi^2 R r^2 \end{aligned}$$