

Important Theorems and Definitions concerning Derivatives

<p>The Definition of the Derivative of a function at a point $(c, f(c))$</p>	$f'(c) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(c + \Delta x) - f(c)}{\Delta x} \right)$ <p>The Alternative definition is</p> $f'(c) = \lim_{x \rightarrow c} \left(\frac{f(x) - f(c)}{x - c} \right)$						
<p>Definition of the Derivative of a Function</p>	$f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$ <p>The Alternative definition is</p> $f'(x) = \lim_{z \rightarrow x} \left(\frac{f(z) - f(x)}{z - x} \right)$						
<p>Notations for derivatives and higher order derivatives</p>	$f'(x) \quad \frac{d}{dx} f(x) \quad D_x f(x) \quad y' \quad \frac{dy}{dx} \quad D_x y$ $f''(x) \quad \frac{d^2}{dx^2} f(x) \quad D_{x^2} f(x) \quad y'' \quad \frac{d^2 y}{dx^2} \quad D_{x^2} y$						
<p>Differentiability Implies Continuity</p>	<p>If a function is differentiable at $x = c$ then the function is continuous at $x = c$.</p>						
<p>Causes of Non-differentiability</p>	<ol style="list-style-type: none"> 1. Discontinuity 2. Sharp turn or cusp 3. Vertical tangent 						
<p>Basic Differential Rules</p>	<ol style="list-style-type: none"> 1. Constant Rule: $D_x [c] = 0$ where c is a constant 2. Power Rule: $D_x (x^n) = nx^{n-1}$ 3. Scalar Rule: $D_x (kf(x)) = kf'(x)$ 4. Sum or difference law $D_x [f(x) \pm g(x)] = f'(x) \pm g'(x)$ 5. Product Law $D_x (f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$ 6. Quotient Law $D_x \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$ 						
<p>The Derivatives of Trig functions</p>	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">1. $D_x (\sin x) = \cos x$</td> <td style="width: 50%;">2. $D_x (\cos x) = -\sin x$</td> </tr> <tr> <td>3. $D_x (\tan x) = \sec^2 x$</td> <td>4. $D_x (\cot x) = -\csc^2 x$</td> </tr> <tr> <td>5. $D_x (\sec x) = \sec x \tan x$</td> <td>6. $D_x (\csc x) = -\csc x \cot x$</td> </tr> </table>	1. $D_x (\sin x) = \cos x$	2. $D_x (\cos x) = -\sin x$	3. $D_x (\tan x) = \sec^2 x$	4. $D_x (\cot x) = -\csc^2 x$	5. $D_x (\sec x) = \sec x \tan x$	6. $D_x (\csc x) = -\csc x \cot x$
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<p>The Chain Rule</p>	<p>If $y = f(u)$ is a differential function of u and $u = g(x)$ is a differential function of x, then $y = f(g(x))$ is a differential function of x and</p> $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad (\text{Leibniz notation})$ <p>or</p> $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \quad (\text{Newton's notation})$
<p>Derivative Rules of logarithms and exponential functions</p>	$\frac{d}{dx} \ln x = \frac{1}{x} \qquad \frac{d}{dx} \log_a x = \frac{1}{x \ln a}$ $\frac{d}{dx} e^x = e^x \qquad \frac{d}{dx} a^x = a^x \ln a$
<p>Derivatives of Inverse Trig Functions</p>	$1. \frac{d}{dx} \text{Arc sin } x = \frac{1}{\sqrt{1-x^2}} \qquad 2. \frac{d}{dx} \text{Arc cos } x = \frac{-1}{\sqrt{1-x^2}}$ $3. \frac{d}{dx} \text{Arc tan } x = \frac{1}{1+x^2} \qquad 4. \frac{d}{dx} \text{Arc cot } x = \frac{-1}{1+x^2}$ $5. \frac{d}{dx} \text{Arc sec } x = \frac{1}{ x \sqrt{x^2-1}} \qquad 6. \frac{d}{dx} \text{Arc csc } x = \frac{-1}{ x \sqrt{x^2-1}}$
<p>The Derivative of an Inverse Function</p>	<p>If $g(x)$ is the inverse function of $f(x)$ and $g(b) = a$</p> <p>then $g'(b) = \frac{1}{f'(a)}$</p>